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Analysis of diameter–density relationships and self-thinning in non-thinned even-aged Scots pine stands

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Abstract

Self-thinning and size–density relationships in non-thinned even-aged Scots pine stands in Spain were studied. Data from nine thinning experimental areas located in natural and artificial stands were used with a total of 23 untreated or control permanent plots. Three different diameter–density expressions were fitted and compared: the Reineke's equation, the modification of this expressions proposed by Zeide [Zeide, B., 1995. For. Ecol. Manage. 72, 265–272], and the self-thinning model developed by Tang et al. [Tang S., Meng, C.H., Wang, Y.H., 1994. For. Ecol. Manage. 70, 67–73]. Results indicate that self-thinning in *Pinus sylvestris* L. follows a concave down line whose slope increases with stand density until it takes as its asymptote the straight line of maximum density. The slope of this straight line obtained for Scots pine stands in Spain is -1.75 . © 2001 Elsevier Science B.V. All rights reserved.

Keywords: *Pinus sylvestris* L.; Self-thinning; Diameter–density relationships; Reineke's equation; Stand Density Index

1. Introduction

Density-induced mortality or self-thinning is the result of competition between the individuals of a stand. This competition takes place if one or more of the resources needed for tree development is insufficient for the needs of individuals and the stand as a whole. The most effective density indexes for the study of the influence of density on self-thinning are those which combine an expression of the size of the average tree (diameter, height, biomass or volume) with the number of trees (Curtis, 1970; Long,

1985). Reineke's expression (Reineke, 1933) and the $-3/2$ power rule of self-thinning (Yoda et al., 1963) are the most commonly used. Nevertheless, the suitability of these two theoretical relations for the purpose of describing the self-thinning process has been called into question by various authors in the last two decades (Drew and Flewelling, 1977; Zeide, 1985; Weller, 1987a; Bredenkamp and Burkhardt, 1990; Lonsdale, 1990; Hynynen, 1993).

In this study we did not try to corroborate or refute the self-thinning rule but to find a good and useful size–density relationship for non-thinned Scots pine stands in Spain. In forest management, knowledge of self-thinning and information about the stand density in which natural mortality increases quickly is useful to determine the optimal thinning regimes for a stand. Moreover, since 'non-thinned stands' will be more frequent, according to the recent tendency of the forest

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management, a more precise knowledge of this natural process is essential.

In order to relate the number of trees of a stand with an expression of the size of the average tree, Zeide (1987) suggests using the square mean diameter (Reineke's expression) instead of the average tree volume or the biomass (Yoda's expression), since the first has a greater capacity to respond to density variation because of being closely correlated with crown width. On the other hand, the square mean diameter can be measured more accurately and easily than volume and biomass and it is the most practical size variable for management.

Reineke postulated that the relation between the number of trees per hectare and the square mean diameter on a double logarithmic scale in untreated stands is a straight line with a constant value of the slope, -1.605 , for all species. This slope or self-thinning rate was calculated for different species in later research, giving slopes varying with the species, and sometimes with age, with values between -2.33 and -1.54 . Thus the idea of the 'straight line' has been called into question (Zeide, 1991; Hynynen, 1993). However, Reineke's equation is still used as stand density index (SDI)² (Bredenkamp and Burkhardt, 1990; Somers and Farrar, 1991; Andenmatten et al., 1997; Bravo et al., 1997; Bravo, 1999). The limitation of this application of Reineke's line is to know the appropriate slope.

Zeide (1987) described the self-thinning process as a concave down curve rather than a straight line ($\ln(N) - \ln(Dg)$). Three phases can be seen in the curve: from young stage till crown closure, when the self-thinning is less than the mortality at maximum competition, it is concave and lies under the straight line of maximum density; in the intermediate phase of the stand's development, it follows the straight line of maximum density; and in aged stands, when the capacity to fill gaps left by dead trees has been lost, the line tends to curve below the straight line. In order to describe this size–density trajectory, the same author proposed a modification of the diameter–density relationship which includes the total height of the stand (Zeide, 1995). Other authors have proposed a

different kind of trajectory for the relation number of trees–diameter, in which once the line of maximum density has been reached, the curve follows it as an asymptote (Smith and Hann, 1984, 1986; Puettmann et al., 1993; Tang et al., 1994).

According to these trajectories the self-thinning straight line or maximum size–density line expresses the upper boundary of all possible size–density relationships, which represents the maximum competition and takes place when stand is fully occupied. We will use the term 'full-stocking' to refer to this maximum competition. Nevertheless, self-thinning may also occur under this maximum line, when stand is 'under-stocked', and can be modelled with a size–density relationships which changes with time (Long and Smith, 1983/1984). The self-thinning model of Tang et al. (1994) is based on the idea that the self-thinning of an under-stocked stand increases with stand density and reaches its maximum at maximum size–density line. The advantage of this approach, which uses full-stocked and under-stocked stands, is that they avoid subjectivity in the selection on the data which represent maximum density (Weller, 1987b).

The goal of this study is to further understand the phenomenon of natural mortality in non-thinned Scots pine stands in Spain and to find the maximum size–density line. Reineke's straight line and the modification proposed by Zeide (1995) are analysed for determining the maximum line in order to use it as density index and the model proposed by Tang et al. (1994) is fitted to the data in order to know the self-thinning trajectory for this species in a Mediterranean environment.

2. Material and methods

2.1. Data

This study made use of data from control plots (non-thinned) of an experimental network of thinning initiated in 1968. The experimental unit of this thinning experiment is the site. There are 10 sites of *Pinus sylvestris* with a total of 105 permanent plots, of which 23 are untreated or control plots situated at nine sites (one site has not control treatment). Plot surface varies between 0.065 and 0.1 ha, and buffers are 10 m wide.

² $SDI = N(Dg/25)^\beta$, where β is the slope of Reineke's line.

Table 1
General information about sites

Site	Plot	Initial density	Site index ^a	Origin	Ages at inventories	No data
SO-1	1	2792	20	Natural	41, 46, 56, 61, 66	5
SO-1	2	2405	23	Natural	41, 46, 56, 61, 66	5
SO-1	3	2435	20	Natural	41, 46, 56, 61, 66	5
SO-2	1	1887	17	Natural	50, 55, 65, 70, 75	5
SO-2	2	2375	20	Natural	50, 55, 65, 70, 75	5
SO-2	3	1737	17	Natural	50, 55, 65, 70, 75	5
SG-1	1	1546	29	Artificial	45, 55, 60, 65, 70	5
SG-1	2	1417	29	Artificial	45, 55, 60, 65, 70	5
SG-1	3	1360	23	Artificial	45, 55, 60, 65, 70	5
SG-2	1	5040	23	Natural	35, 45, 50, 55, 60	5
SG-2	2	6170	23	Natural	35, 45, 50, 55, 60	5
SG-2	3	5960	20	Natural	35, 45, 50, 55, 60	5
BU-1	1	3571	29	Natural	41, 51, 56, 61	4
BU-1	2	3230	29	Natural	41, 51, 56, 61	4
BU-1	3	3110	29	Natural	41, 51, 56, 61	4
M-1	1	1940	29	Artificial	38, 43	2
BU-2	1	1710	29	Artificial	22, 27, 32	3
BU-2	2	1620	29	Artificial	22, 27, 32	3
BU-2	3	1540	29	Artificial	22, 27, 32	3
SG-3	1	2700	29	Artificial	22, 28	2
M-3	1	2260	23	Artificial	39, 44	3
M-3	2	2420	26	Artificial	39, 44	3
M-3	3	2020	26	Artificial	39, 44	3

^a According to the Rojo and Montero (1996) quality sites.

The Scots pine stands selected for the study are located in the mountain areas of the Sistema Central and Sistema Ibérico, belonging to two different provenance's areas for this species in Spain (provenance's areas 'Sierra de Guadarrama' and 'Montaña de Burgos-Soria' according to Catalán et al. (1991)). Stands are of natural and artificial regeneration and cover different quality sites (site indexes from 17 to 29 at 100 years), initial densities and age classes (Table 1). More information about the data is available in Río (1998).

In each plot, diameter at breast height of every tree, total height in a sample of 30 trees covering all diameter classes, and total height in another sample of 10 dominant trees was measured. Inventories were performed generally every 5 years, but in some cases the interval was longer, 8 or 10 years, for reasons beyond our control. From the 23 control plots, 91 values were obtained, analysed and summarised (Table 2). Possible autocorrelation among residuals due to the use of repeated measures had been analysed by lag-residual plots.

2.2. Reineke's equation

Reineke's equation for fully stocked stands relates the number of trees of a stand with their square mean diameter:

$$\ln(N) = \alpha - \beta \ln(Dg) \quad (1)$$

where N is the number of trees per hectare, Dg is the square mean diameter, α is the intercept of the straight line and β is the slope of this line.

In order to carry out this fitting and to interpret the result as a line of maximum size–density, it is necessary to be certain that the stands analysed are

Table 2
Description of data

Variable	Min	Max	Mean	S.D.
Age	20	75	50.80	12.73
Trees (ha)	1034	6170	2187	337
Dg (cm)	9.48	27.08	17.58	3.65
H (m)	6.60	22.03	12.85	1.14

full-stocked or at maximum density (Weller, 1987b). For this reason, after a data inspection, we have ignored: (a) data where the mortality annual rate of the plot is under 1%; (b) data from the first inventory of each plot, since we do not know if there was previous density-dependent mortality, i.e. if the stands were in a self-thinning phase (Hynynen, 1993); and (c) data concerning mortality due to exceptional causes such as the 1996 snowstorm. Applying these criteria, the 91 data from the 23 control plots were reduced to 40 for the study of Reineke's straight line.

2.3. Size–density relationship of Zeide (1995)

Zeide (1995) proposed a modification to Reineke's equation considering that the slope would not be constant, but changing with age. Laasenaho and Koivuniemi (1990) also showed that the linear fit between average diameter at crown base and number of trees (on the log–log scale) is better than the Reineke's equation (between square or average diameter at breast height and number of trees). Assuming that diameter at crown base is related to both diameter at breast height and distance between crown base and breast height³, and that this distance is proportional to total height, results of Laasenaho and Koivuniemi (1990) can be transformed into a new relationship between number of trees, diameter and height:

$$\ln(N) = \alpha - \beta \ln(Dg) + \chi H \ln(Dg) \quad (2)$$

where H is the average total height and χ is a new parameter representing the variation of the slope.

Although Zeide (1995) applied his equation to various stands, thinned and unthinned, in this paper it was applied only to unthinned stands. In order to explain self-thinning in fully stocked stands, and to establish a practical density index, this modification of Reineke's relation (Eq. (2)) was fitted to the same data as Eq. (1).

2.4. Self-thinning model of Tang et al. (1994)

The self-thinning model developed by Tang et al. (1994) relates the number of trees with the square

³ $D_c = D^{1-ms}$, where D_c is the diameter at crown base, D is the diameter at breast height, s is the distance between D_c and D , and m is a parameter (Zeide, 1995).

mean diameter, and assumes that in a non-thinned stand, which has not reached the maximum density, the self-thinning rate increases with stand density index. This hypothesis can be formulated as the equation:

$$\frac{d \ln N(t)}{d \ln Dg(t)} = -\beta \left(\frac{S(t)}{S_f} \right)^\gamma \quad (3)$$

where $N(t)$ is the number of trees per hectare at time t , $Dg(t)$ is the square mean diameter at time t , $S(t)$ is Reineke's density index at time t , S_f is Reineke's density index for maximum density, β is the slope of the line of maximum density, and γ is a parameter dependent on the species, defined as the self-thinning index.

When the stand is fully stocked the stand density index is S_f and the self-thinning rate becomes constant and equal to the slope of the line of maximum density. Thus, this model can be applied to under and fully stocked stands.

Solving Eq. (3) and describing stand density index $S(t)$ as:

$$S(t) = N(t) \left[\frac{Dg(t)}{D_o} \right]^\beta \quad (4)$$

where D_o is the standard base diameter (equal to 25 cm), the model expression to be fitted can be written as:

$$N(t) = \frac{S_f}{\left[(Dg(t)/25)^{\beta\gamma} + \delta \right]^{1/\gamma}} \quad (5)$$

where δ is the integration constant dependent on an initial value (N_1, Dg_1).

The non-linear model was fitted to the data from all the control plots of the thinning experiment (62 values), taking as initial values the number of trees per hectare, and the mean square diameter of the first inventory of each plot. Data used are from stands which were undergoing density induced mortality because under these stands did not growth shrubs due to crown closure (Long and Smith, 1983/1984), there was social differentiation and data concerning mortality due to the disturbances have been eliminated. The three parameters (S_f , β and γ) are constant for the same species and are estimated by non-linear regression.

Linear regressions were fitted using the REG procedure of SAS (SAS Institute Inc., 1988). For the non-linear regression the DUD algorithm from NLIN procedure was performed (Ralston and Jennrich, 1978). Initial values of S_f and β parameters were obtained from the estimates of the Eq. (1). The value previously determined by Tang et al. (1994) for *P. resinosa* Ait. was retained as the initial value of γ . In order to obtain a constant error variance, the regression has been weighted with the inverse of the square mean diameter variance (Huang et al., 1992).

3. Results

The results of the Reineke's equation (Eq. (1)) fitted to fully stocked Scots pine stands are presented in Table 3. The adjusted correlation coefficient obtained in the linear regression is $R^2_{adj}=0.892$, with a squared mean error of 0.0184. Fig. 1 shows the fitted straight line and the data used in the regression.

The slope obtained (-1.829) is significantly different from the one given by Reineke (-1.605) at the 95% confidence level. Stand density index (SDI) calculated with the estimated slope is 1188 trees per hectare.

The modified model proposed by Zeide (Eq. (2)) was fitted to the same data (Table 4). When height was included in the relationships between number of trees and diameter, the adjusted correlation coefficient increases ($R^2_{adj}=0.938$) and the squared mean error decreased to 0.0104.

In order to adjust these models, some data from untreated plots were eliminated. Density-dependent mortality in untreated plots, even if not very high, is not altogether absent, thus indicating that self-thinning also occurs in stands which have not reached their

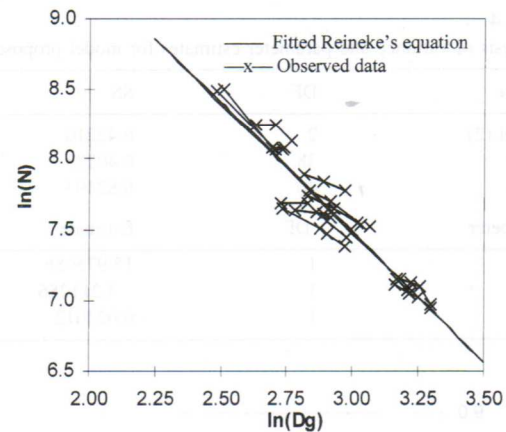


Fig. 1. Predicted (straight line) and observed data ($-x-$) from the Reineke's equation fitted for non-thinned even-aged Scots pine stands. Note: (slope= -1.83). Data from stands which were not certain at maximum density were removed.

maximum possible density. Fig. 2 shows the self-thinning lines of all the available control plots, eliminating only the data of the last inventory of the SG-2 site, which reflects a exceptional snowstorm damage. This figure shows how non-thinned stands of Scots pine grow. If the stand develops from a low initial density, mortality in the first years is very low, but once the stand approaches maximum density, it begins to increase. It can also be seen that different stands tend to converge to the same asymptotic line. According to these results (Fig. 2), data would fit better to the second type of trajectory described, where the line of maximum density is the asymptote of self-thinning.

The results of the non-linear model (Eq. (5)) proposed by Tang et al. (1994) are shown in Table 5 (estimates of the parameters together with their standard deviation and confidence interval). The weighted mean square error obtained in the regression is 0.082

Table 3
Analysis of variance and parameter estimates for Reineke's expression (Eq. (1))

Source	DF	SS	MS	F	P
Model (1)	1	6.10722	6.10772	332.123	0.0001
Error	39	0.71721	0.01839		
Total	40	6.8249			
Parameter	DF	Estimate	Standard error	T for Ho:	Prob> T
α	1	12.968935	0.29430403	44.066	0.0001
β	1	-1.829518	0.10038929	-18.224	0.0001

Table 4

Analysis of variance and parameter estimates for model proposed by Zeide (1995) (Eq. (2))

Source	DF	SS	MS	F	P
Model (2)	2	6.42210	3.21105	302.907	0.0001
Error	38	0.40283	0.01060		
Total	40	6.82493			
Parameter	DF	Estimate	S.E.	T for Ho: $a=0$	Prob> T
α	1	15.975686	0.59562966	26.822	0.0001
β	1	-3.211256	0.26492899	-12.121	0.0001
χ	1	0.023012	0.00422573	5.446	0.0001

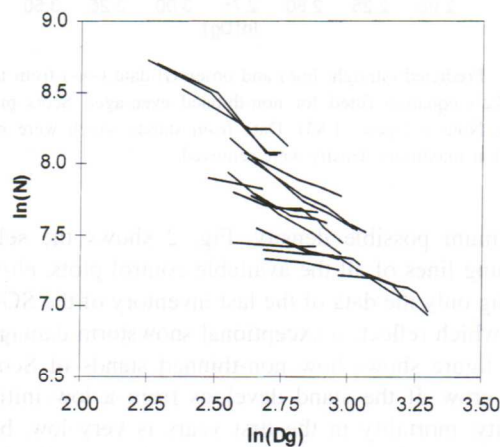


Fig. 2. Observed square mean diameter (D_g)–density (N) relationships or self-thinning lines in non-thinned even-aged Scots pine stands. Note: Data belong to fully and under-stocked stands.

and the relative error is 7.29%. The lag-residual graphs for this model are presented in Figs. 3 and 4 in order to analyze the autocorrelation among residuals that often appears in time series. These graphs do not show a strong autocorrelation among residuals.

The value of the β (1.750) parameter of model (5) differs from that obtained by adjusting Reineke's

Table 5

Parameter estimation for self-thinning model proposed by Tang et al. (1994) (Eq. (5))

Parameter	Estimate	Standard error	95% confidence intervals
S_f	1444.820	47.197	1350.378 1539.261
β	1.750	0.061	1.628 2.873
γ	2.394	0.243	1.908 2.881

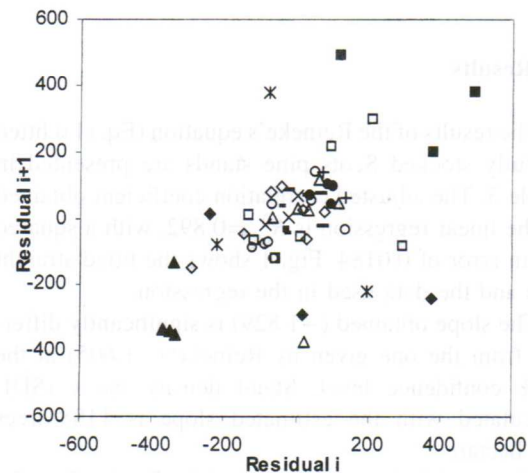


Fig. 3. Lag-residual graph for analysis of temporal correlation in the adjusted model proposed by Tang et al. (1994) (Eq. (5)). Data with the same symbol are from the same plot.

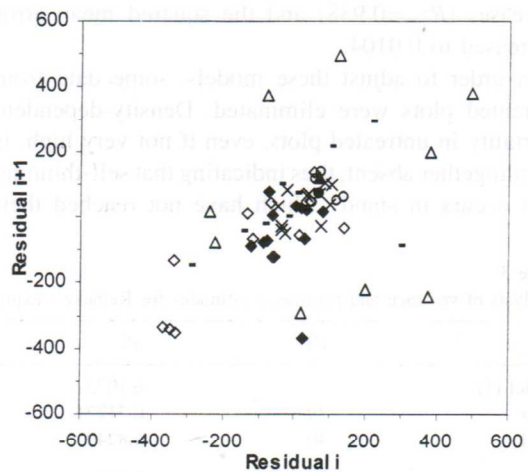


Fig. 4. Lag-residual graph for analysis of spatial correlation in the adjusted model proposed by Tang et al. (1994) (Eq. (5)). Data with the same symbol are from the same site.

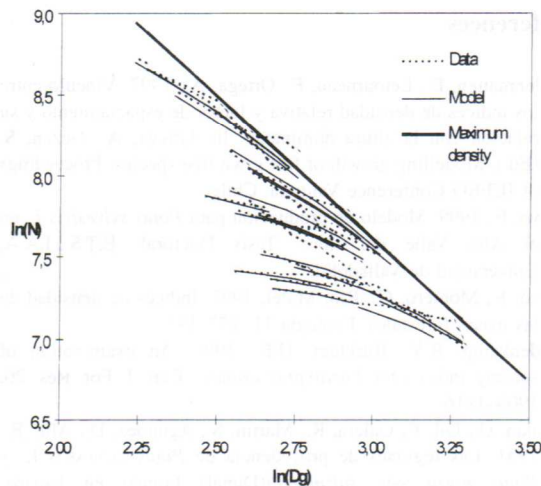


Fig. 5. Fitted and observed evolution of self-thinning lines and straight line of maximum density (slope = -1.75) obtained with the model proposed by Tang et al. (1994).

straight line (1.823), although not significantly at 5%. The stand density index calculated from Reineke's straight line previously fitted (1188 trees/ha) is lower than the value obtained from the Tang model (1445 trees/ha), and even is beyond the confidence interval of S_T .

Fig. 5 shows the real and simulated evolution of self-thinning, including the maximum density line obtained with the last model. It can be seen that most of the data used in the adjustment are below the maximum density line.

4. Discussion

The results show that in the stands under study self-thinning occurs in under-stocked stands (Fig. 2). For this reason, it is important to be careful when the self-thinning rate is used as stand density index (SDI). It is necessary to know if the slope of the Reineke's line is well estimated. The slope obtained using the data of non-thinned stands is significantly different from the one proposed by this author, confirming again the dependence of the slope on the species (Zeide, 1985; Bredenkamp and Burkhardt, 1990; Puettmann et al., 1993). Nevertheless it is very similar to the slope obtained for Scots pine stands by Rojo and Montero (1996) in the Sistema Central in Spain (-1.836) and

by Hynynen (1993) for the same species in Finland (-1.844). This reflects a constant value for this species in spite of changing the environmental features. The model proposed by Tang et al. (1994) gives a smaller slope, but all the values mentioned above fall within its confidence interval (Table 5).

The intercept of Reineke's fit, 12.97, is lower, but not significantly different to that obtained by Rojo and Montero (1996), 13.13. Hynynen (1993) found an slightly lower intercept which varied with site quality, but in the present study there are not enough data for studying this dependence. Two arguments can explain the lower value of maximum density in Finland: it could be a sign of a dependence of the productivity on the geographical area, similar to the findings of Debell et al. (1989) for *P. taeda* L.; and it could also be due to the different nature of the data in both studies. The first argument would be agree with the study of Río (1998), in which Scots pine yield tables, in different countries, were contrasted, finding that the productivity of Spanish stands is higher than in Finland.

Using Eq. (2) as a size–density relationship for maximum density stands, the fit is statistically better than with Eq. (1). In Zeide's equation the maximum density fit is a curve which depends on the average height, a new parameter in his model. This suggests a dependence on age or site quality. Nevertheless, Bredenkamp and Burkhardt (1990) report no evidence of such dependence in stands with high competition. They account for the variations found by other authors on the nature of their data, probably below maximum density, as in the study of Zeide (1995). Moreover, Sterba and Monserud (1993) indicate that it is difficult to find maximum density stands, since stands do not frequently reach that density due to the many factors intervening in natural mortality. Consequently, and taking into account the Reineke's expression simplicity, it is advisable to use Reineke's density index (SDI) with the new exponent as an approximate measure of stand density.

Considering the evident difficulty of assessing if a stand is at highest density, and given that there is natural mortality below this density, it is most efficient to use models which use under and fully stocked stands in studies of self-thinning, avoiding subjectivity in the selection of data (Puettmann et al., 1993; Smith and Hann, 1984). Thus, results from model (5) are more trustworthy, since the data used are certainly

valid for the fitting, feature which is lost when fitting only maximum density stands (Eqs. (1) and (2)).

The maximum density obtained with Eq. (1) is below the confidence interval of parameter S_f , while the maximum density for the Sistema Central given by Rojo and Montero (1996), 1369 trees/ha, falls within it. Thus the asymptote of model (5) is probably closer than Eq. (1) to the real maximum density line. This fact corroborates the difficulty of choosing appropriate data to fit the self-thinning in fully stocked stands.

Following the fit of the last model, Scots pine self-thinning trajectory is concave down until it reaches the maximum density lineal asymptote. Data did not allow verification of Zeide's hypothesis, which proposes that in the old growth phase the self-thinning line moves away from the maximum density line. In order to study this feature it would be desirable to have data from old age untreated stands, since it seems probable that with increasing age stands lose their ability to close gaps created by natural mortality.

5. Conclusions

From the study of Reineke's equation in Scots pine stands in Spain, a slope higher than the value proposed by Reineke (1933) is obtained. This is relevant since a good estimate of the slope of the maximum density line is necessary in order to use Reineke's equation as a density index. The equation proposed by Tang et al. (1994) performed well to the self-thinning process of untreated Scots pine stand up to 75 years old. This indicates that the density-dependent mortality or self-thinning of Scots pine stands occurs as well in under stocked stands, increasing with the stand density until the density reaches its maximum value of 1444. The slope of this maximum density straight line is -1.75 . We propose the use of the new exponent to estimate Stand Density Index in Scots pine stands in Spain in order to achieve the density of these stands.

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