

Spatial prediction of diameter distribution models

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Abstract

The prediction of the diameter distribution of a stand is of great interest to forest managers for the evaluation of forest resources and scheduling the future silvicultural treatments. In the present paper, we present a geostatistical approach for the prediction of diameter distributions. The advantage of the proposed method over traditional prediction systems is that it is possible to estimate the diameter distribution at locations, where no secondary variables are measured.

The Weibull and the Chaudhry–Ahmad probability density functions were fitted by maximum likelihood to the diameter distribution of 176 plots of pure, even aged stands of *Pinus pinaster* Ait. located in the province of Segovia of central Spain. Data were taken from the Second National Forest Inventory of Spain. The spatial “behaviour” of the parameters of the density functions was studied by variogram analysis and the results showed that all parameters were spatially correlated to some extent. Kriging was used for the interpolation of parameters of the diameter distributions over the study area. Cross-validation was performed separately for every parameter. Results showed that the Weibull function gave smaller relative bias in the prediction of the diameter sum. Additionally, the likelihood ratio was used to compare the fit of the density functions. This ratio was computed twice: firstly when parameters were estimated by maximum likelihood and secondly, when parameters were predicted by kriging-cross-validation. The Chaudhry–Ahmad function fitted better the actual diameter distributions when maximum-likelihood was used for parameter estimation. Nevertheless, the fit of the Weibull was much better when kriging was used for parameter prediction.

The prediction of the diameter distribution was made using data from the National Forest Inventory, therefore predictions should be used for regional planning and estimation of forest resources at this spatial scale (national). For forest management purposes at a local scale, data sources taken at fine spatial scales should be preferred. The spatial discontinuities that the forest compartments introduce during kriging interpolation are finally discussed and some solutions are proposed. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

The prediction of the distribution of trees in a stand according to their diameter at breast height (DBH) is

of major importance to forest managers, since it can be used to forecast the range of products which might be expected from a stand (Rennolls et al., 1985). Modeling the DBH distribution is usually a two-stage process: a probability density function (pdf) is fitted to the actual diameter distributions at the first stage, and then the pdf parameters are regressed against other, easily measured or a priori known stand attributes (parameter prediction method). Explanatory

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variables such as the mean stand diameter or the stand age are used to predict parameters of the pdf (Rennolls et al., 1985; Maltamo et al., 1995; Maltamo, 1997). Other methods for predicting diameter distributions include parameter recovery methods (Shiver, 1988; Khatouri and Dennis, 1990; Knowe, 1992) and distribution independent approaches (Tang et al., 1997). The most commonly used probability density function for modelling diameter distributions is the Weibull function (Bailey and Dell, 1973). However, other functions such as the beta have been also proposed (see for example Maltamo et al., 1995).

Often repeated measurements from permanent plots are available, but researchers avoid systematically using them with ordinary least squares regression (OLS) during the parameter prediction. Instead, it is preferred to use only one measurement of the same plot in order to avoid the problem of data auto-correlation (Rennolls et al., 1985). The statistical problems that arise when data from repeated measurements are used with OLS have been identified and partially solved with the use of the generalised linear model (Ferguson and Leech, 1978; West and Vanclay, 1995).

Contrary to the temporal, the spatial auto-correlation of parameters of diameter distribution models has been ignored until now. Forest ecosystems are the most complex and organised of their terrestrial counterparts. Their stability and succession depends greatly on their organisation in space. Various studies at small spatial scales (within a stand) have shown that the size of a tree in a stand depends on the size of its neighbours as result of competition for water, light and nutrients (Szwagrzyk, 1992; Moeur, 1993; Szwagrzyk and Czerwczak, 1993). Biondi et al. (1994) found that variation of stem size and increment in an old-growth ponderosa pine forest was spatially structured. At larger scales, spatial auto-correlation has been found for the site-index (Hock et al., 1993), the tree total height (Samra et al., 1989) and the timber volume (Holmgren and Thureson, 1997; Gunnarsson et al., 1998).

Spatial auto-correlation of data used in ecological studies can be a problem or a solution, depending on the objectives and the methodology used (Legendre, 1993). Matérn (1994) revises the methods and concepts that have been used in forestry for spatial data analysis. Auto-correlation in space complicates the classical statistical analyses because observations

are assumed to be independent (temporal auto-correlation of repeated measurements produces similar statistical problems). Geostatistics on the other hand provide a means to use the spatial auto-correlation for prediction purposes. Variogram analysis and kriging are the essential geostatistical tools, described at various textbooks (Journel and Huijbregts, 1978; Goovaerts, 1997; Armstrong, 1998). Stationarity (a fundamental assumption made during kriging), has to be assumed in order to minimise and quantify the prediction error.

In the present paper we have used the Weibull and the Chaudhry–Ahmad pdf in our effort to model the diameter distribution of pure, even-aged stands of maritime pine (*Pinus pinaster* Ait.). On a second stage we examined the spatial structure of their parameters by variogram analysis and we used kriging for spatial prediction. Finally, some comparisons have been made in order to choose the best pdf for prediction purposes.

2. Material

Data came from the second Spanish National Forest Inventory. Sampling was realised on a square grid of 1 km in size. The sampling scheme was circular plots of varying radius, thus, the inclusion of a tree in the sample was a function of its DBH and its distance from the centre of the plot. The minimum measured diameter varied with the radius of the plot being 75, 125, 225 and 425 mm for radius 5, 10, 15 and 25 m, respectively. All trees in the plot were measured for DBH and total height, among other characteristics. Up to six trees in every plot were selected for more specific measurements such as diameter at different heights or crown diameter.

The plots that we included in the analysis are located in the “Tierra de Pinares” of central Spain. The natural maritime pine formations of the region are grown on a very poor soil consisted mainly of dunes. In the past years, the primary objective of the forest management was the resin production, while timber was considered of secondary importance. After one century of intensive forest management, the resulting stands are of low densities (150–200 trees/ha).

Only plots that included more than 10 trees of maritime pine were used for the analysis (176 plots).

Trees that were not measured for DBH because their diameter was smaller than the minimum measured had to be added before the density functions were fitted.

3. Methods

The bi-parametric Weibull and the Chaudhry–Ahmad functions were fitted to each plot. The Chaudhry–Ahmad pdf (Chaudhry and Ahmad, 1993) for the random variable x ($x > 0$) is given by:

$$f_A = 2\left(\frac{a}{\pi}\right)^{-1/2} \exp\left[-\left(\sqrt{a}x - \sqrt{b_A}\frac{1}{x}\right)^2\right] \quad (1)$$

where b_A and a are the parameters of the probability function. The mode is found to be:

$$\mu_0 = \left(\frac{b_A}{a}\right)^{1/4} \quad (2)$$

The substitution of $b_A = a\mu_0^4$ to (1) leads to:

$$f_A = 2\left(\frac{a}{\pi}\right)^{-1/2} \exp(2a\mu_0^2) \times \exp\left[-a\mu_0^2\left(\left(\frac{x}{\mu_0}\right)^2 + \left(\frac{\mu_0}{x}\right)^2\right)\right] \quad (3)$$

where μ_0 and $a\mu_0^2$ are, respectively, the location and shape parameter of the probability function.

Maximisation of the log-likelihood function of (1) with respect to parameters a and b_A leads to the maximum likelihood estimators (MLEs) $\hat{a}$ and $\hat{b}_A$:

$$\hat{a} = \hat{b}_A(m'_{-2})^2 \text{ and } \hat{b}_A = [2(m'_{-2}m'_2 - 1)m'_{-2}]^{-1} \quad (4)$$

where

$$m'_2 = \frac{1}{n} \sum_{i=1}^n x_i^2 \text{ and } m'_{-2} = \frac{1}{n} \sum_{i=1}^n \frac{1}{x_i^2} \quad (5)$$

The two-parameter Weibull function for the random variable x ($x > 0$) is given by:

$$f_w = \left(\frac{c}{b}\right) \left(\frac{x}{b}\right)^{(c-1)} \exp\left[-\left(\frac{x}{b}\right)^c\right] \quad (x > 0, b > 0, c > 0) \quad (6)$$

where b and c are, respectively, the scale and shape parameter of the probability function. Substituting

$\theta = 1/b^c$ in (6) the log-likelihood function is given by:

$$L = \log \left\{ \prod_{i=1}^n \theta c x^{(c-1)} \exp(-\theta x^c) \right\} \quad (7)$$

Maximisation of Eq. (7) can only be achieved iteratively. Instead of maximising the log-likelihood, we minimised the negative log-likelihood, using a quasi-Newton algorithm written in S for S-plus (S-Plus 4.5., 1998). The procedure was repeated two times for every plot, using different starting values in order to avoid convergence at local minimum.

The comparison of the fit of the density functions was done at this stage using the natural logarithm of the likelihood ratio:

$$\ln L(x) = \ln \left\{ \frac{\prod_{i=1}^n f_A(x_i, \vartheta_A)}{\prod_{i=1}^n f_W(x_i, \vartheta_W)} \right\} \quad (8)$$

where $L(x)$ is the likelihood ratio, n the number of trees in the plot, f_A and f_W the Chaudhry–Ahmad and the Weibull density function, x_i the vector of observations, and ϑ_A and ϑ_W the vectors of estimated parameters for the Chaudhry–Ahmad and the Weibull density, respectively. Both the probability functions have two independent parameters, therefore according to the Akaike's Information Criterion (AIC), the Chaudhry–Ahmad fit is better than the Weibull when $\ln L(x) > 0$.

The null hypothesis that the average log-likelihood ratio is equal to zero was checked with a randomisation procedure. A total of five hundred permutations were performed by placing randomly at the denominator of Eq. (8) the Weibull or the Chaudhry–Ahmad likelihood and computing the average log-likelihood ratio. The null hypothesis was rejected when the average value was smaller (higher) than the 0.01 (0.99) empirical percentile. This procedure is equivalent to a simple paired-samples t -test being more effective than this because no assumptions for normality and independence need to be made.

On a second stage, we computed the experimental variograms for the parameters of the density functions, for the mean plot diameter and for the coefficient of diameter variation. Theoretical variogram models were fitted to the experimental ones and interpolation was performed by kriging assuming stationarity.

Predictions made by kriging were cross-validated. Each one of the plots was deleted and the kriged parameter was predicted using the rest of the observations.

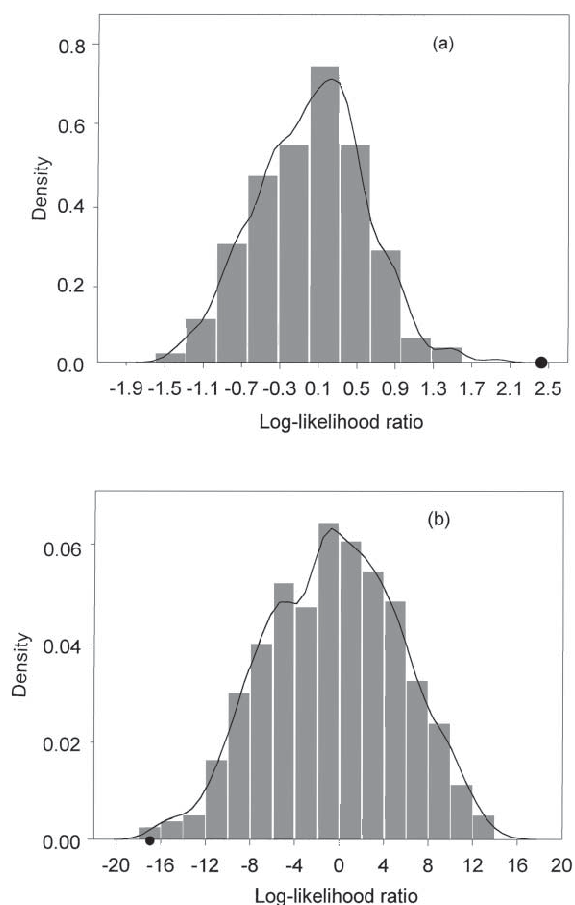
The kriged average error, kriged mean square error, the mean as well as the variance of the standardised error were used for the comparison between the density functions (Armstrong, 1998). Furthermore we computed once more the natural logarithm of the likelihood ratio, using the parameters that resulted from cross-validation and we used the same randomisation procedure, to test the null hypothesis that the average log-likelihood ratio is equal to zero.

Although, the log-likelihood ratio is useful for the selection and comparison of two distributions, it does not provide any information about the difference between the actual and the predicted distribution. Error indices that have been used for this reason, include the bias or the mean square prediction error between the observed and predicted powers of diameter sums, over the hole distribution or over 5 cm diameter classes (Maltamo et al., 1995; Maltamo, 1997). In this study, we computed the relative bias and root mean square error (RMSE) for the diameter sum, using the formulas presented by Maltamo (1997). Since kriging is an exact interpolator, we had to use the cross-validation parameter estimates, in order to compute the predicted diameter sum. Cross-validation is widely used in geostatistics for model criticism and comparison (Goovaerts, 1997; Kitanidis, 1997). Some of the drawbacks of using cross-validation for model criticism have been reported in the geostatistical literature. The slope of the linear regression between actual and re-estimation scores, for example, can never reach unity due to the Lagrange multiplier (for ordinary kriging) (Armstrong, 1998), while a more serious problem is that the cross-validation errors reflect the worst error scenario when dealing with data on a regular grid, since the removal of a single datum renders the local sampling intensity unusually low. Furthermore, when removing a datum that is located at the border of the study area, then kriging is called for extrapolation and not for interpolation. The idea of cross-validation is to produce sample data configurations that mimic the conditions under which the estimation procedure will actually be used (Isaaks and Srivastava, 1989). Therefore, cross-validation results are valid only for the performance comparison between the Weibull and Chaudhry–Ahmad predictions, but they are not representative of the errors during the normal prediction.

4. Results

4.1. Models for parameters

The algorithm that we used for the estimation of the Weibull parameters could not converge at nine plots, therefore comparisons between the models are based on the remaining 167 plots. The mean likelihood ratio computed using the ML parameter estimates was equal to 2.47 which was found to be significantly different than zero since the results of five hundred permutations (Fig. 1a) showed that this value was much higher than the empirical 0.99 percentile



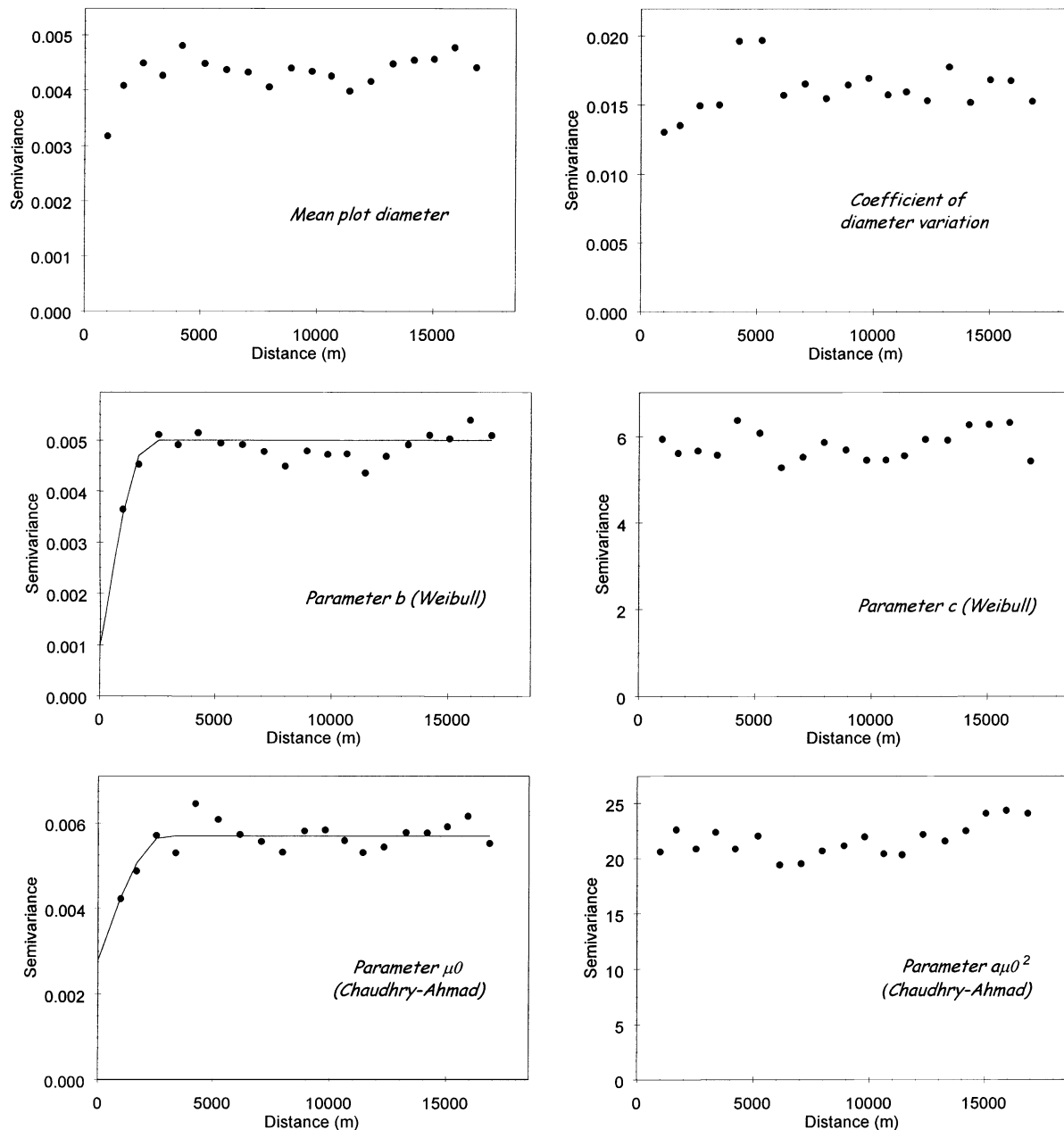


Fig. 2. Omnidirectional experimental variograms for parameters of interest. The solid lines represent the spherical variogram models that were fitted to the experimental ones.

(1.44). Therefore, at this stage the fit of the Chaudhry–Ahmad was better than that of the Weibull function.

Spatial correlation is evident for the mean plot diameter, the coefficient of variation as well as

for parameters b and μ_0 (Fig. 2), but not for the shape parameters of the distribution functions which are non-linearly and negatively correlated with the coefficient of variation (Fig. 3a–c). We tried various

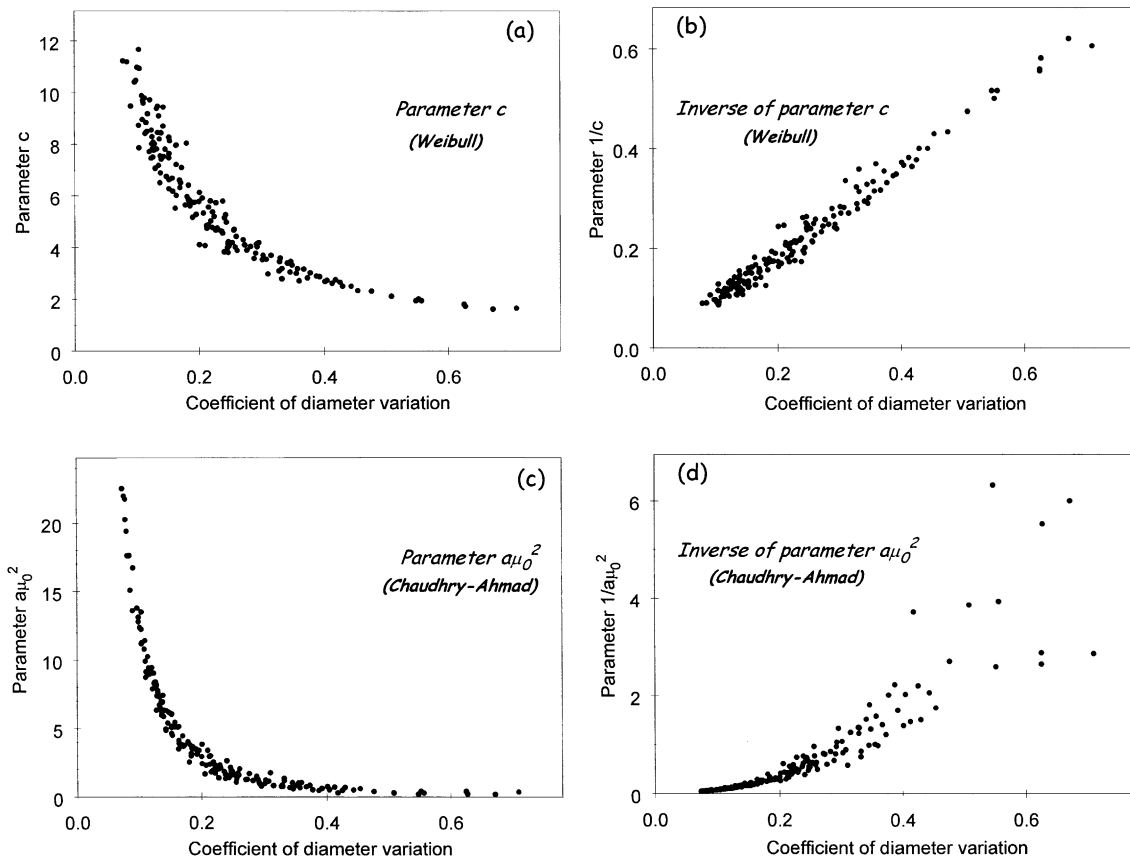


Fig. 3. Shape parameters and their inverse with respect to the coefficient of diameter variation.

transformations of the shape parameters, pursuing that one that would result in a better variogram. The inverse of the shape parameter was finally selected. As it can be seen in Fig. 3b and d this transformation made

the relationship between the Weibull shape parameter and the coefficient of variation a linear and close to the 1:1. The variograms of the inverse of the shape parameters show that auto-correlation is present

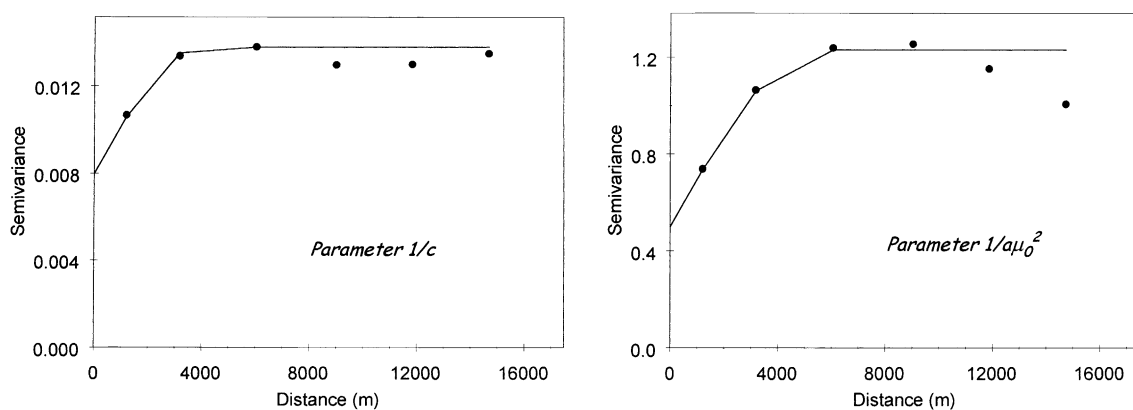
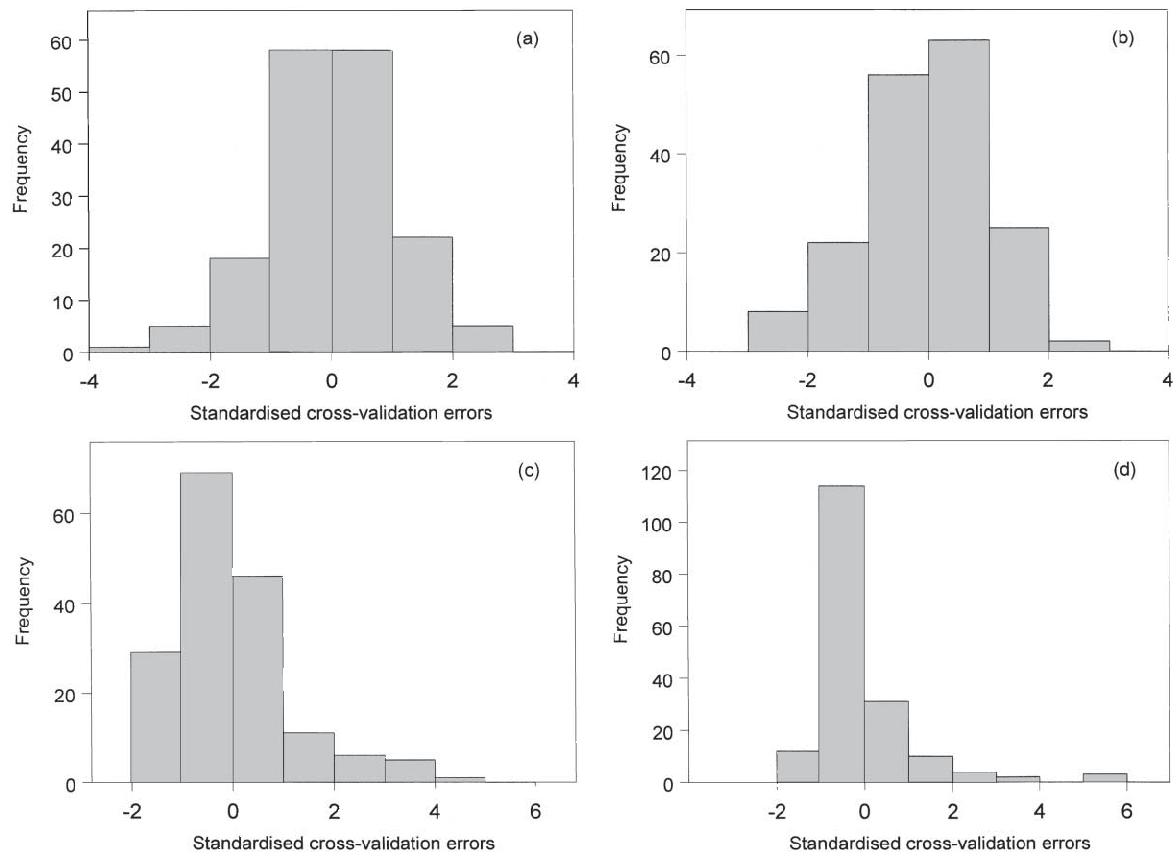


Fig. 4. Omnidirectional experimental variograms and spherical models for the inverse of the shape parameters.

(Fig. 4). Spherical theoretical variograms were used to model the experimental ones (Table 1). Directional experimental variograms were also computed for all the variables of interest, but no significant anisotropies were detected.

4.2. Model comparisons

Cross-validation showed that the standardised prediction errors made during kriging have a close to normal distribution for parameters b and μ_0 , while



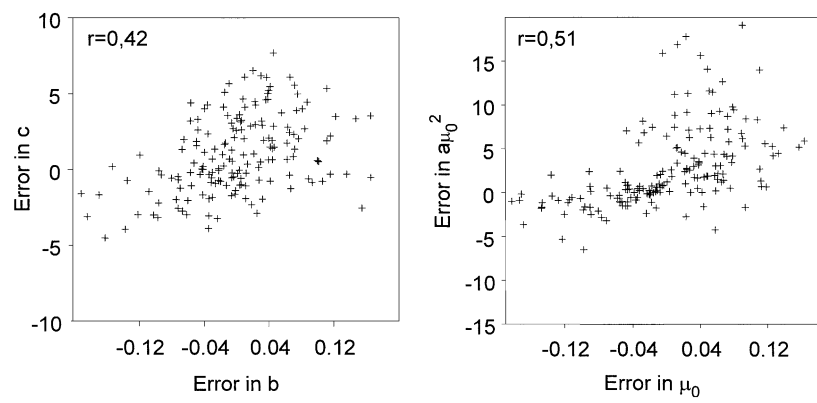


Fig. 6. Correlation between cross-validation errors for the parameters of the Weibull and the Chaudhry–Ahmad probability functions.

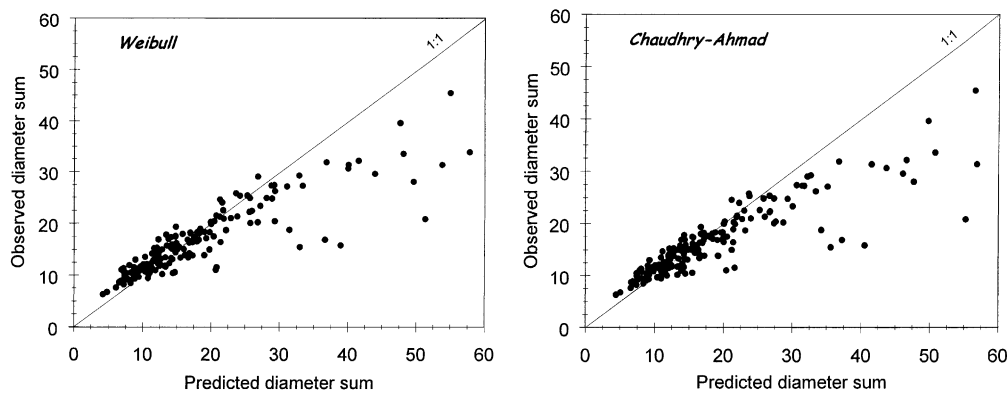


Fig. 8. Observed diameter sum of all trees in a plot vs. the predicted (from kriging cross-validation) diameter sum, for the Weibull and the Chaudhry–Ahmad probability functions.

the distribution of the standardised error for the shape parameters is positively skewed (Fig. 5). No big differences were detected in the cross-validation statistics between parameters b and μ_0 , but in the case of the shape parameters, we found that predictions of the Chaudhry–Ahmad parameter were more biased and less precise compared to the Weibull shape parameter (Table 2).

Errors during the interpolation of parameters are correlated to some extent (Fig. 6). The Pearson's correlation coefficient was lower for the errors of the Weibull parameters ($r = 0.42$) than for the Chaudhry–Ahmad ($r = 0.51$).

The re-calculation of the log-likelihood ratio showed that the fit of the Weibull is better than the Chaudhry–Ahmad when parameters are predicted with kriging. This time the average ratio over the 167 plots was equal to -16.95 and the 0.01 empirical quartile -14.02 leading to the conclusion that the null hypothesis is rejected (Fig. 1b). Fig. 7, shows the location of each plot with respect to the values of the log-likelihood ratio. The quadrant with the lowest number of plots is the fourth, meaning that only in 13 plots the fit of the Chaudhry–Ahmad was worse for ML and better for kriging. However, in 56 plots (quadrant II) the fit was better for the Chaudhry–Ahmad for ML, but worse for kriging.

Regarding the errors in the prediction of the diameter sum of the plot, we found the Weibull function to be less biased compared to the Chaudhry–Ahmad. However, the relative RMSE was practically the same for both functions (Table 3). Fig. 8 shows the observed

versus the predicted sum of diameters for the 167 plots that were used for comparisons. Both graphs indicate that predictions are conditionally biased due to the smoothing effect of kriging. It is not surprising to see that overestimation occurs in plots with large diameter sum (the inverse would be expected for kriging). This is due to the inverse relationship between the diameter sum and the location parameter of the diameter distribution (usually the diameter sum is large in young and dense plots where the location parameter of the diameter distribution is small).

5. Discussion

Stand characteristics such as the mean stand diameter or the diameter coefficient of variation present spatial auto-correlation in the study area. Actually this is a not unexpected phenomenon in managed forests where the range of spatial correlation depends on the length of the forest compartments (Mandallaz, 2000). Usually in a forest inventory, when the mean distance among sampling plots is smaller than the length of the forest compartments, then data will be spatially correlated. Spatial correlation of stand attributes has been investigated in previous studies. Hock et al. (1993) and Payn et al. (1999) found spatial correlation for the site index and used it for interpolation. Samra et al. (1989) used the spatial dependence for the tree height to produce maps of forest productivity of plantations of *Melia azedarach*. Other authors have found spatial correlation directly

for the stand timber volume and used it for spatial prediction (Holmgren and Thuresson, 1997; Gunnarsson et al., 1998).

Much work has been done on the prediction of parameters of diameter distribution models using stand attributes such as the stand age or the mean diameter (Kilikki et al., 1989; Maltamo et al., 1995; Maltamo, 1997). Nevertheless, the parameter's spatial auto-correlation has been ignored during the parameter prediction. In classical statistics, it is assumed that observations are independent so that each sampling unit offers one degree of freedom. If parameters are indeed spatially auto-correlated this assumption is violated since new observations do not offer one full degree of freedom (Legendre, 1993). The resulting OLS coefficients are unbiased, but they are not minimum variance estimators and their standard errors are underestimated (Cook and Pocock, 1983). A possible overcome to this problem is to use only those sampling units whose distance is bigger than the range of the variogram. If this is not possible, the construction of prediction models should use the general linear model, which allows for spatially correlated errors. The spatial covariance has to be estimated and subsequently use generalised least squares assuming that residuals are spatially correlated to some extent (Bailey and Gatrell, 1995). Mandallaz (2000) has proposed a simple least squares method for the estimation of the spatial covariance in forest inventories.

Finally, the use of different probability functions can be a solution. The parameters of the functions that we used in this paper that control the location of the distribution on the x -axis (i.e. parameters b and μ_0) are directly related to the mean stand diameter which presents spatial correlation. However, parameters of the beta function for example, are not usually spatially auto-correlated at this scale because they are a function of the minimum and the maximum plot diameter. The range of spatial correlation of those variables is usually much smaller than the sampling interval of a national forest inventory (1 km in the case of the Spanish inventory).

The interpolation of parameters of the distribution functions was not without problems. Correlated errors were found during interpolation. Spatial models that are based on interpolated parameters have to study and quantify the effect of correlated errors (Phillips and

Marks, 1996). Correlation of the interpolation errors is probably due to the correlation of parameters themselves. When both the shape and the location parameters are underestimated, then middle diameter classes are underestimated and the extremes overestimated. On the contrary, when parameters are overestimated, then the extreme diameter classes will be underestimated and the middle ones overestimated.

The comparisons that have been made between the Chaudhry–Ahmad and the Weibull density function showed that the former fits better the actual diameter distributions when ML is used for parameter estimation. Nevertheless, the Weibull function gave better fits when kriging was used for parameter prediction. Obviously, the spatial prediction of the Weibull parameters is more accurate. The reasons could be the better spatial “behaviour” of the Weibull parameters or the less correlated errors. Most important though seems to be the nature of the functions itself. The α parameter of the Chaudhry–Ahmad density cannot be predicted because no spatial correlation has been found. Instead, the shape ($\alpha\mu_0^2$) and the location (μ_0) parameters have to be predicted independently in order to compute the α parameter. Thus, the errors made in prediction of both parameters are multiplied when prediction for α is required, influencing negatively the total accuracy.

The approach that we advocate in this paper for the prediction of the stand diameter distribution has a great advantage over traditional stand projection systems since it can give a prediction at a location, where no secondary information is available. This property can be proved especially useful when predictions are required over large spatial scales (national or regional for example). Other stand characteristics such as tree height or the stand density could also be interpolated and integrated into a spatial prediction system for forest resources evaluation to be made.

Nevertheless, geostatistical predictions over different scales have to be avoided. When data from National Forest Inventories are used, then predictions should be used for regional planning and estimation of forest resources at this spatial scale (national). For forest management purposes at a local scale, data sources taken at finer spatial scales should be preferred.

Spatial stochastic modelling of diameter distribution models was proved to be an efficient modelling

strategy. Nevertheless, the intensive forest management that has been applied in the study area introduces an inevitable spatial discontinuity when moving from one compartment to another. Therefore, interpolation close to the borders of the compartments is problematic especially when the sampling is not very intense.

The problem of forest compartments should be studied in future works. Other authors have tried some solutions. Holmgren and Thuresson (1997) for example, have used combined estimates of image data and kriging interpolation. When a sufficient number of sample plots exists different analysis for every age class in a forest can be performed (Gunnarsson et al., 1998; Dessard, 1999), a method which is presently called kriging within strata (Goovaerts, 1997). Full co-kriging can be another solution (Wackernagel, 1998). Usually parameters of the diameter distribution models are correlated with the mean diameter. Therefore, when the mean diameter is known it can be used as an additional covariate.

Some other alternatives are the simple kriging with varying local means, kriging with an external drift or colocated co-kriging (Goovaerts, 1997). The disadvantage of these methods is that the secondary variable must be known at all locations being estimated.

Space-time modelling finally is in our opinion the most promising approach (Houllier and Pierrat, 1992; Stein et al., 1998; Kyriakidis and Journel, 1999). This relatively new technique uses time as an additional dimension. In forestry, the stand age used as the time-dimension will resolve the problem of forest compartments, while combined estimations in space and time can be given. The major obstacle of using spatio-temporal geostatistics in forest-growth and -yield modelling is the estimation of the temporal covariance since a trend will always be present in the data.

6. Conclusions

Parameters of diameter distribution models are spatially correlated in the study area. Generally, in managed forest the range of spatial correlation is a function of the length of the forest compartments. This type of auto-correlation should be taken into account when predicting parameters of diameter distribution models.

If a spatial prediction approach for parameters is to be used, then the Weibull or the Chadhry–Ahmad function should be used for modelling, because their parameters are related to stand attributes that are usually spatially structured. In that case, before any spatial prediction is realised, care should be taken in order to overcome the problem of forest compartments. Finally, the scale at which the data are collected determines up to a certain degree the usefulness of the predictions. Data from national forest inventories will give predictions to be used for national or regional planning and evaluation of forest resources.

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