

Base-age invariant cork growth model for Spanish cork oak (*Quercus suber* L.) forests

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Abstract A cork growth model for Spanish cork oak forests was developed using data from 432 cork samples. Ten dynamic equations derived from the generalised algebraic difference approach (GADA) were considered for analysis, and both numerical and graphical methods were used to compare alternative models. All of the equations are base-age invariant and directly estimate accumulated cork thickness in complete years at any moment of the cork rotation. The fittings were done using the nested iterative method of the stochastic regression approach. The GADA formulation derived by Krumland and Eng (2005) from the Richards model by considering a_1 and a_3 as related to site productivity was finally selected. The cork growth model developed in this study allows the total accumulated cork thickness in complete years to be estimated in the fourth year of cork rotation with close to 80% reliability for any debarking period, increasing to nearly 90% from the eighth year onwards.

Keywords Cork growth · Cork thickness · Dynamic equation · Generalised algebraic difference approach · Nested iterative method

Introduction

Cork production constitutes a primary source of income from cork oak stands prevailing in pre-coastal and coastal

regions of the Mediterranean Basin (Montero and Cañellas 1999). Spain is the second major cork producing nation with 510,000 ha of cork oak stands (23% of the world's total) and an annual production of 110,000 t (32% of the world's total) (Natural Cork Quality Council 1999). Although the main use of these stands is cork production, they are also efficiently exploited for other uses which include hunting, cattle grazing (mainly Iberian pigs), firewood, or biological and landscape diversity.

The profits obtained from these stands are closely related to cork quality and its use in the production of natural cork stoppers, which are the most valuable product and mainstay of the cork industry. Natural cork provides the best closure for fine wines with a proven tradition of reliability and excellence. Cork quality depends on three main characteristics: cork thickness, cork porosity and the presence of defects such as insect galleries or occasional wood inclusions (Montero and Cañellas 1999). Any one of these factors may lead to the raw material being rejected by the stopper industry and, consequently, result in a loss of revenues (wine stoppers account for approximately two-thirds of cork revenues). Cork thickness defines the usability and the value of the cork for industrial purposes. Cork planks with a thickness over 27 mm are suitable for the production of stoppers, and the best yield is obtained with a thickness between 27 and 33 mm (González-Adrados et al. 2000). Cork planks with lower thickness values are used for less-valuable products such as discs and articles made from agglomerated cork, while planks thicker than 40 mm result in wastage and therefore, less economic yield.

Despite its economic and industrial importance, cork growth has seldom been modeled. Vieira (1950) proposed a second degree polynomial model that related accumulated cork growth at the end of the debarking period to cork age.

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A similar approach embraces the cork growth model developed by Montero et al. (1994), in which the authors assess the relationship between cork growth and cork age varying from 9 to 13 years and present a second degree polynomial model without intercept. González-Adrados et al. (2000) developed an equation for predicting total cork thickness at the time of debarking where the independent variable was cork thickness one year before stripping. With these cork growth models, the decision on whether to delay the cork harvest would be taken the year before debarking. Finally, the cork growth model developed by Tomé et al. (2001) used the difference form of the Lundqvist–Korf model which leaves a_1 (asymptote parameter) as the free parameter. This model is included as a sub-model in the SUBER model, a management oriented growth and yield model developed in Portugal for open cork oak woodlands.

Cork growth is complex and variable. This large variability in cork thickness is associated with unobservable tree attributes, probably to do with genetics or microsite rather than social status (Sánchez-González et al. 2006). Growth variables for which the changes modeled over time are additionally dependent on an initial condition may be modeled using self referencing functions (Northway 1985), which use current states of the considered phenomena (in this case accumulated cork thickness in complete years at a certain number of complete years after cork harvest) to determine their past or their future states. This makes the methods developed to fit these types of models especially suitable for modelling cork growth. A great variety of methods have been employed in the construction of site index models. According to Clutter et al. (1983), three general methods can be used for developing site index curves: the guide curve approach, the parameter prediction method and the algebraic difference approach. The latter of the three allows the development of models based on dynamic equations, with the advantage, among others, of greater flexibility as well as being more parsimonious (e.g. Cieszewski and Bailey 2000; Cieszewski 2001, 2003; Krumland and Eng 2005).

Dynamic equations can be derived by using base-age invariant methods such as the algebraic difference approach (ADA) or the generalized algebraic difference approach (GADA). The ADA was first proposed by Schumacher (1939) and cited in Bailey and Clutter (1974) who gave the name to the method and proposed the base-age invariant parameter estimation technique based on covariance analysis. The ADA method is founded on the replacement of a base-model site-specific parameter with its initial-condition solution. The main limitation of this approach is that all models thus derived are either anamorphic or have single asymptotes (Bailey and Clutter 1974; Cieszewski and Bailey 2000). The GADA first proposed by Cieszewski (1994)

and cited by Cieszewski and Bailey (2000), is a generalization of the ADA and builds on the findings of Bailey and Clutter (1974) and García (1983).

The objective of this study is to develop a base-age invariant cork growth model for Spanish cork oak forests that would be suitable as: (1) a tool which enables us to decide whether a cork harvest should be delayed in order to obtain cork planks suitable for stopper production; and (2) a module of the management oriented growth and yield model which is being developed by the authors in the CIFOR-INIA for cork oak forests in Spain (Sánchez-González et al. 2005b).

Materials and methods

Study area and data

The Natural Park of “Los Alcornocales” (Fig. 1), with an area of 170.025 ha is one of the most important cork producing areas in the country and can be considered representative of higher density Spanish cork oak forests (Sánchez-González et al. 2005a) as opposed to woodlands which are associated with lower densities. The area has a mild Mediterranean climate with cool humid winters and warm-dry summers; the mean annual temperature is about 16–18°C and the annual precipitation between 1,000 and 1,400 mm (depending on altitude). Precipitation is mainly concentrated between autumn and spring, originating a dry period in summer (De Benito 2000). The soils are cambisols and luvisols (FAO 1998), which are fairly well developed.

Data for this study were collected from cork samples obtained from 432 trees in 22 circular plots of 1,257 m² established by the Forest Research Center (CIFOR-INIA) in the Natural Park. All plots were established in regularly stocked stands covering a wide range of age and site conditions. During cork stripping, a cork sample with dimensions 20 × 20 cm² was taken at breast height from each tree. Ring width for each sample was measured using a linear positioning digitizer tablet (LINTAB), and the data obtained were saved and processed with the aid of the TSAP software (1996).

Descriptive statistics of tree characteristics from cork samples are displayed in Table 1. Table 2 shows the cork growth variability in the year of the cork extraction

Only data from complete growth years were considered in the model (3,745 observations from 432 trees), the first and last half years of a debarking period were not considered. Therefore, the variable modeled is the accumulated cork thickness after t complete years of growth. Summary statistics of the observations used in this study were calculated by complete growth years (Table 3).

Fig. 1 Distribution of *Quercus suber* L. in Spain and location of the studied region



Cork growth modelling

The GADA can be used to derive the same models as those derived from the ADA but with the advantage that the base equations can be expanded according to various theories on growth characteristics (e.g. asymptote, growth rate), allowing more than one parameter to be site-specific, and permitting the derivation of more flexible dynamic equations (Cieszewski and Bailey 2000; Cieszewski 2001, 2002, 2003). It includes the ability to simulate concurrent polymorphism and multiple asymptotes. Krumland and Eng (2005) summarized the methodology described by Cieszewski and Bailey (2000) for developing GADA models for site index curves into five steps:

1. The essence of GADA is to formulate an explicit base model describing the relationship between size, time, and site: $Y = f(t, X)$ (where X is the unobservable site variable) by identifying all the parameters in the base

equation that potentially change for different levels of site productivity and then reformulating the base equation by replacing these parameters with functions of X (one unobservable independent variable that describes site productivity as a summary of management regimes, harvesting intensity, soil conditions, and climatic factors) as well as new global parameters. In this way, the initially selected two-dimensional site equation ($Y = f(t)$) expands into an explicit three-dimensional site equation ($Y = f(t, X)$).

2. Solve the resulting GADA formulated model for X .
3. Form a specific solution for X in terms of equivalent initial conditions.
4. Substitute the initial conditions, collect terms, simplify as much as possible, and produce the final GADA based model with either the same number or less parameters than the initial explicit site productivity equation.

The generalized algebraic difference approach (GADA) (Cieszewski and Bailey 2000) was used to derive the dynamic equation for cork growth modelling.

The implementation of a base-age invariant cork growth model in the sense of Bailey and Clutter (1974) involves the use of dynamic equations along with the estimation of the model parameters using a base-age invariant method. The base age invariant method assumes that the data measurements always contain environmental as well as measurement errors (both on the left and the right sides of the model) which must be modelled (Cieszewski 2003). After considering the different approaches available for estimating such parameters, we opted for the Iterative Evaluation Approach, which uses nested iterative procedures to fit the global and site-specific parameters (Tait et al. 1988; Cieszewski et al. 2000).

Table 1 Characterization of the fitting data set

Variable	No of observations	Mean	Minimum	Maximum	Standard deviation
cc	3,745	2.65	0.24	9.35	0.97
cb	432	26.34	12.20	53.70	5.95
w	432	20.87	4.00	217.50	19.06
sh	432	1.87	0.78	5.27	0.74
d_{uc}	432	32.08	14.64	90.40	10.20
cr	432	9.67	9	12	1.16

cc Annual cork growth (mm); cb cork thickness (mm); w tree cork weight (kg); sh stripped height (m); d_{uc} diameter at breast height under cork (cm); cr cork rotation (years)

Table 2 Characterization of the variability in each cork extraction

Debarking year	Cork rotation	Number of plots	<i>n</i>	Variable	Mean	Minimum	Maximum	Standard deviation
2000	9	15	1,968	cc	2.77	0.47	6.35	0.83
				cb	24.45	12.20	40.64	4.80
2001	9	1	112	cc	4.08	1.53	6.87	1.26
				cb	35.19	22.80	41.41	5.51
2002	9	2	480	cc	2.63	0.74	8.51	1.13
				cb	25.04	14.93	42.55	5.88
2002	11	2	470	cc	2.39	0.55	6.23	0.92
				cb	27.97	19.89	42.29	5.74
2003	12	2	715	cc	2.48	0.24	13.15	1.26
				cb	30.37	17.50	53.70	6.23

n Number of observations;
cc annual cork growth (mm);
cb cork thickness (mm)

Candidate functions

A total of ten dynamics models were formulated using the GADA approach. With regard to the general notational conventions in all model development, $a_1, a_2, \dots, a_n$ are used to denote parameters in base models and $b_1, b_2, \dots, b_n$ are used for global parameters in subsequent GADA-based models. All GADA-based models have the general implicit form of $Y = f(t, t_0, Y_0, b_1, b_2, \dots, b_n)$. Table 4 shows the different dynamic functions used as candidate cork growth models in the parameter estimation process.

Models C4, C8 and C10 were formulated considering two parameters to be site specific. Model C4 was derived by Barrio-Anta et al. (2006) from the Lundqvist–Korf (Lundqvist 1957; Korf 1939) function and model C8 by Krumland and Eng (2005) from the Richards (1959) function, both of them by expressing the asymptote as an exponential function of X and the shape parameter (a_2 in C4 and a_3 in C8) as a linear function of the inverse of X .

Table 3 Total accumulated cork thickness in complete year statistics given in millimeters by complete growth years

Complete growth year	No of observations	Mean	Minimum	Maximum	Standard deviation
1	432	3.68	0.82	9.93	1.36
2	432	6.41	2.14	13.25	1.88
3	432	9.21	3.53	16.42	2.45
4	432	12.16	5.14	21.65	3.11
5	432	14.94	5.61	27.37	3.75
6	432	17.80	7.10	31.54	4.28
7	432	20.71	9.19	36.18	4.78
8	432	23.27	11.10	40.15	5.23
9	112	24.83	15.62	37.83	5.22
10	112	26.99	16.77	45.71	5.72
11	65	29.60	17.37	51.86	6.09

Model C10 was developed by Cieszewski (2002) from the Hossfeld (1822) model by substituting a_1 for a constant plus the unobserved site variable X , and a_2 by a_2/X .

The rest of the dynamic models were derived by considering only one parameter to be site specific, in this case, GADA is equivalent to ADA. The first three models were developed from the Lundqvist–Korf (Lundqvist 1957; Korf 1939) function and models C5–C7 from the Richards (1959) function. Model C9 was proposed by McDill and Amateis (1992) considering parameter a_2 in the Hossfeld (1822) model as site specific.

Parameter estimation

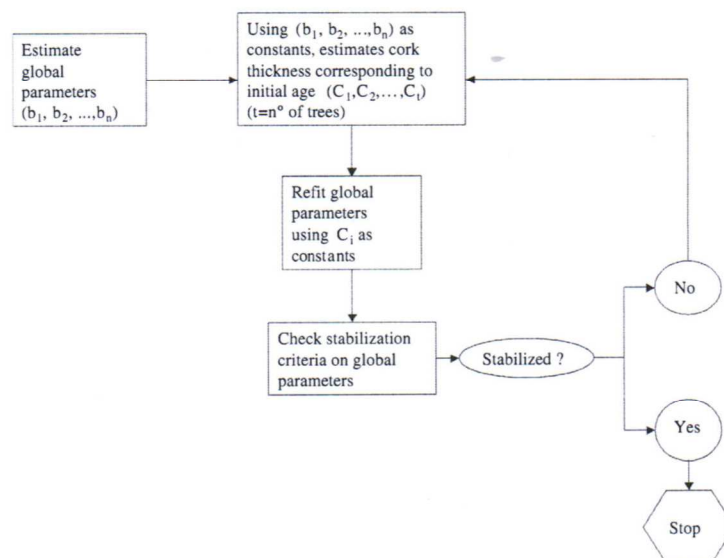
The base-age invariant parameter estimation technique used was the iterative evaluation approach. This method uses nested iterative procedures to fit the global and site-specific parameters. It was first proposed by Tait et al. (1988) and was further developed by Cieszewski et al. (2000). The initial conditions specified must be identical for all the measurements belonging to the same tree. The initial age can be arbitrarily selected, although an age of zero is not acceptable. The iterative procedure can be summarized as follows: (1) the global parameters are fitted using the observed accumulated cork thickness in complete years corresponding to the initial age; (2) using estimates of the global parameters as constants, the accumulated cork thickness in complete years corresponding to the initial age is estimated for each tree; (3) the estimated accumulated cork thicknesses in complete years corresponding to the initial age for each tree are used as constants and the global parameters are refitted. This procedure is repeated until the global parameter is stabilized, as illustrated in Fig. 2 (adapted from Cieszewski et al. 2000).

In order to address the occurrence of serial correlation resulting from repeated measurements taken from the same tree, which is a situation common to cork samples, the error

Table 4 Candidate equations for cork growth modelling

Citation source and base size-time equation	Solution for X with initial values (t_0 , Y_0)	Dynamic equation	Function code
Amaro et al. (1998) $Y = Xe^{-a_2 t^{-a_3}}$	$X_0 = Y_0(e^{-a_2 t^{-a_3}})^{-1}$	$Y = Y_0 e^{\left(\frac{1}{\frac{1}{b_3} - \frac{1}{b_1}}\right)}$	C1
Bailey and Clutter (1974) $Y = a_1 e^{-X t^{-a_3}}$	$X_0 = -\ln(Y_0 a_1^{-1}) t_0^{a_3}$	$Y = b_1 (Y_0 b_1^{-1})^{\left(\frac{Y}{Y_0}\right)^{b_3}}$	C2
Amaro et al. (1998) $Y = a_1 e^{-a_2 t^{-a_3}}$	$X_0 = -\log_{t_0}(-\ln(Y_0 a_1^{-1}) a_2^{-1})$	$Y = b_1 \left(\sqrt[1+a_3]{b_2^{-1}} \ln(b_1 Y_0^{-1}) \right)^{-1}$	C3
Barrio-Anta et al. (2006) $Y = e^X e^{-(b_1 + b_2 X^{-1}) t^{-a_3}}$	$X_0 = \frac{1}{2} t_0^{-b_3} \left(b_1 + t_0^{b_3} \ln(Y_0) \pm \sqrt{4b_2 t_0^{b_3} + (-b_1 - t_0^{b_3} \ln(Y_0))^2} \right)$	$Y = e^{X_0} e^{-(b_1 + b_2 X^{-1}) t^{-b_3}}$	C4
Pienaar and Shiver (1980) $Y = X(1 - e^{-a_2 t})^{\frac{1}{1-a_3}}$	$X_0 = Y_0 \left((1 - e^{-a_2 t})^{\frac{1}{1-a_3}} \right)^{-1}$	$Y = Y_0 \left(\frac{1 - e^{-b_2 t}}{1 - e^{-b_2 Y_0}} \right)^{\frac{1}{1-b_3}}$	C5
Clutter et al. (1983) $Y = a_1 (1 - e^{-X t})^{\frac{1}{1-a_3}}$	$X_0 = -\ln(1 - (Y_0 a_1^{-1})^{\frac{1}{b_1}}) t_0^{-1}$	$Y = b_1 \left(1 - \left(1 - (Y_0 b_1^{-1})^{\frac{1}{b_1}} \right)^{\frac{t}{t_0}} \right)^{b_3}$	C6
Amaro et al. (1998) $Y = a_1 (1 - e^{-a_2 t})^{\frac{1}{1-a_3}}$	$X_0 = -(\ln(1 - e^{-a_2 t_0}) - \ln(Y_0 a_1^{-1})) (\ln(Y_0 a_1^{-1}))^{-1}$	$Y = b_1 \left(\frac{1 - e^{-\frac{t}{t_0} (1 - e^{-b_2 t_0})}}{1 - e^{-b_2 Y_0}} \right)^{\frac{1}{1-b_3}} Y_0^{\frac{b_3 + b_1}{1-b_3}}$	C7
Krumland and Eng (2005) $Y = e^X (1 - e^{-(b_1 + b_2 X^{-1}) t})^{\frac{1}{1-a_3}}$	$X_0 = \frac{1}{2} \left((\ln(Y_0) - b_2 L_0) \pm \sqrt{(\ln(Y_0) - b_2 L_0)^2 - 4b_3 L_0} \right)$ with $L_0 = \ln(1 - e^{-b_1 t_0})$	$Y = Y_0 \left(\frac{1 - e^{-b_1 t}}{1 - e^{-b_1 Y_0}} \right)^{\frac{b_3 + b_1}{b_3}}$	C8
McDill and Amateis (1992) $Y = \frac{a_1}{1 + X t^{-a_3}}$	$X_0 = t_0^{-b_3} (a_1 Y_0^{-1} - 1)$	$Y = \frac{b_1}{1 - \left(1 - \frac{b_1}{Y_0} \right) \left(\frac{Y}{Y_0} \right)^{b_3}}$	C9
Cieszewski et al. (2002) $Y = \frac{b_1 + X}{1 + (b_2 X^{-1}) t^{-a_3}}$	$X_0 = \frac{1}{2} \left((Y_0 - b_1) \pm \sqrt{(Y_0 - b_1)^2 + 4b_2 Y_0^{-b_3}} \right)$	$Y = b_1 \left(1 - (1 - b_1 Y_0^{-1}) (t_0 t^{-1})^{b_3} \right)^{-1}$	C10

Fig. 2 Iterative evaluation approach (adapted from Cieszewski et al. 2000)



term was modelled using a first-order autoregressive error structure (AR(1)). This error structure is adequate for equally spaced measurement times, the correlations being monotonically decreasing function of elapsed time (Gregoire et al. 1995; Zimmerman and Núñez-Antón 2001). The AR(1) expands the error terms as follows:

$$e_{ij} = d_1 \rho_1 e_{ij-1} + \varepsilon_{ij} \quad (1)$$

where e_{ij} is the j th ordinary residual on the i th individual (i.e., the difference between the observed and the estimated accumulated cork thickness in complete years of tree i at age measurement j), $d_1 = 1$ for $j > 1$ and is zero for $j = 1$, ρ_1 is the first-order autoregressive parameter to be estimated and ε_{ij} is now the error term under conditions of independence and no heteroscedasticity. To test for the presence of autocorrelation graphs representing residuals versus lag-residuals from previous observations for each tree were examined visually.

The iterative evaluation approach including the AR(1) error structure was programmed using a SAS macro which uses the SAS/ETS procedure PROC MODEL (SAS Institute Inc. 2004), allowing for dynamic updating of the residuals. The Marquardt algorithm, which is most useful when the parameter estimates are highly correlated (Fang and Bailey 1998; Parresol 2001), was used for fitting the model.

Model selection

The evaluation of the models was based on numerical and graphical analyses of the residuals. Four statistics were examined: the bias, which reflects the deviation of the model with respect to observed values; the root mean

square error (RMSE), which analyses the precision of the estimates; the coefficient of determination for nonlinear regression (R^2) and Akaike's information criterion differences or delta AIC (Δ_i). The expressions may be summarised as follows:

$$\text{bias} = \frac{\sum (y_i - \hat{y}_i)}{n} \quad (2)$$

$$\text{RMSE} = \sqrt{\frac{\sum (Y_i - \hat{Y}_i)^2}{n - p}} \quad (3)$$

$$R^2 = r_{y_i \hat{y}_i}^2 \quad (4)$$

$$\Delta_i = \text{AIC}_i - \text{AIC}_{\min} \quad (5)$$

where Y_i and $\hat{Y}_i$ are the measured and estimated values of the dependent variable, respectively, n is the total number of observations used to fit the model, and p is the number of model parameters, $r_{y_i \hat{y}_i}$ is the correlation coefficient for a linear regression between the observed and predicted values of the dependent variable (Ryan 1997), AIC_i is the value of Akaike's information criterion for model i , $\text{AIC}_{\min}$ is the minimum of the different AIC_i values. As a rule of thumb, a $\Delta_i < 2$ suggests substantial evidence for the model, values between 3 and 7 indicate that the model has considerably less support, whereas a $\Delta_i > 10$ indicates that the model is very unlikely (Burnham and Anderson 2002).

Another important step in evaluating the models was to perform graphical analyses of the residuals, noting in particular the appearance of the fitted curves overlaid on the trajectories of the cork growth for each individual.

The characterisation of the model error was analysed simultaneously using a validation technique. Since an independent validation data set was not available, a self-sufficient resampling type validation method (multiple

cross-validation) was used. Thus, the models were fitted n times, leaving out each tree once, so that the number of fittings was equal to the number of trees. The residuals obtained in the multiple cross-validation were used to calculate the bias, RMSE and the model efficiency (MEF, equivalent to the R^2 in the fitting phase). Although this approach does not really constitute a method for model validation (Vanclay and Skovsgaard 1997; Pretzsch et al. 2002; Huang et al. 2003), it has been used as an additional criterion for selecting the best model (Myers 1990).

At any given year during the rotation period, a cork growth model should be able to predict the final accumulated cork thickness in complete years at the end of that rotation period and hence the importance of evaluating the reliability of such predictions. For this purpose, different complete growth years and their corresponding observed accumulated cork thicknesses were used to estimate accumulated cork thickness at other complete growth years for a certain debarking period as well as for each tree using the selected model. The results were compared to the observed values for each tree and the relative prediction error was then calculated as follows:

$$RE(\%) = \frac{\sqrt{\frac{\sum (Y_i - \hat{Y}_i)^2}{n-p}}}{\bar{Y}} \quad (6)$$

where Y_i , $\hat{Y}_i$ and $\bar{Y}$ are the measured, estimated and average values of the dependent variable, respectively, n is the total number of observations used to fit the model, and p is the number of model parameters.

Results

The results obtained by fitting the candidate equations are shown in Tables 5, 6. Models C2, C3, C4, C6 and C7 did not meet the convergence criterion. As this circumstance persists when the number of maximum iterations is increased, the convergence criteria are decreased, or the initial parameter values are changed, these models will not be considered further. All the parameters were found to be significant at the 5% level except parameter b_3 in model C5 (Table 5). All of the models tested described the data adequately (Table 6), all of them explained about 98.8% of the total variance in the fitting and validation phases and provided a band of residuals scattered randomly around zero with homogeneous variance and no trends. The graphs showing bias and root mean square error in accumulated cork thickness for complete year estimations by the number of complete years after cork harvest were analysed (Fig. 3), revealing a very similar behavior for all the models. In Fig. 4, it can be appreciated that the trends in residuals as a function of lag1-residuals within the same tree disappear

Table 5 Parameter estimates, corresponding standard errors and P values for the models analysed

Model	Parameter	Estimate	Approx. standard error	t value	Approx. $P > t $
C1	b_2	22.88	3.75	6.11	<0.0001
	b_3	0.04	0.01	5.68	<0.0001
	ρ	0.69	0.01	52.17	<0.0001
C5	b_2	0.04	0.00	11.98	<0.0001
	b_3	-0.02	0.01	-1.90	0.0579
	ρ	0.68	0.01	51.22	<0.0001
C8	b_1	0.04	0.00	12.23	<0.0001
	b_2	0.57	0.08	6.84	<0.0001
	b_3	1.86	0.37	5.07	<0.0001
	ρ	0.68	0.01	51.12	<0.0001
C9	b_1	186.71	13.17	14.17	<0.0001
	b_3	0.97	0.01	115.50	<0.0001
	ρ	0.68	0.01	51.54	<0.0001
C10	b_1	94.99	42.38	2.24	0.0250
	b_2	2957.05	1495.30	1.98	0.0481
	b_3	0.99	0.01	93.04	<0.0001
	ρ	0.68	0.01	51.18	<0.0001

after correction for autocorrelation using a first-order autoregressive error structure (left column). No higher-order autocorrelation was observed (right column). The fitted curve shapes were not significantly altered as compared to the models fitted without considering such correction.

The bias and root mean square error of cork growth is analysed for each complete growth year of the cork rotation in Fig. 3. The poorest behavior corresponds to the 11th year due to the reduced number of observations for that complete growth year. However, even here, the values are less than 2.25 mm, which corresponds to the level of precision used by the cork industry when classifying cork planks.

The GADA formulation derived by Krumland and Eng (2005) from the Richards model by considering a_1 and a_3

Table 6 Values of the goodness-of-fit statistics for fitting and cross-validation phases for the analysed models

Model	Fitting phase				Cross-validation phase			
	Bias	RMSE	R^2	ΔAIC	Bias	RMSE	R^2	ΔAIC
C1	-0.01	0.877	0.9886	70	-0.01	0.868	0.989	72
C5	-0.00	0.861	0.9887	7	-0.00	0.861	0.989	9
C8	0.01	0.860	0.9888	0	0.01	0.860	0.989	0
C9	0.00	0.861	0.9887	15	0.00	0.861	0.989	17
C10	0.00	0.861	0.9887	8	0.00	0.861	0.989	9

Fig. 3 Bias and root mean square error (RMSE) in accumulated cork thickness for complete year estimations for models C1, C5, C8, C9 and C10 in relation to number of complete years after cork harvest

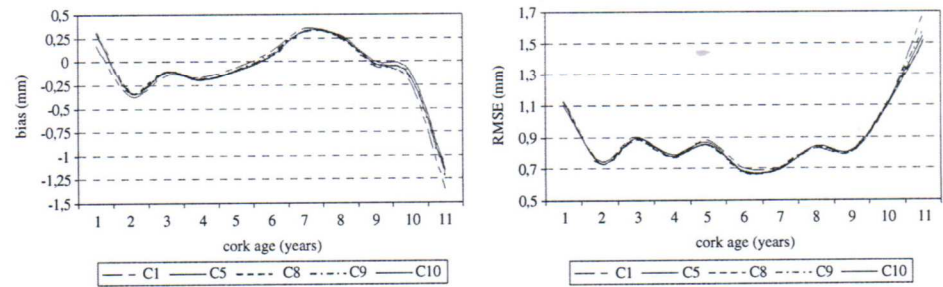
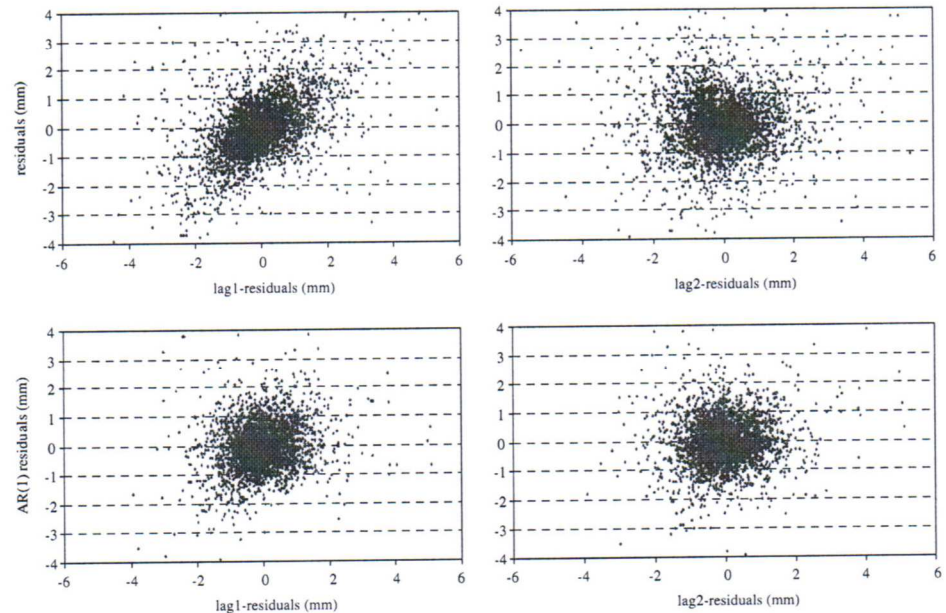


Fig. 4 Residuals versus lag1-residuals (left column) and lag2-residuals (right column) for model C8 fitted without considering the autocorrelation parameters (first row), and using a first-order autoregressive error structure (second row)



as related to site productivity (C8) resulted in the minimum bias and RMSE values and indicated the statistical significance of every parameter at the 0.0001 level. Consequently this model was selected. Figure 5 shows fitted curves for accumulated cork thickness in complete years of 15.75, 22.5, 29.25, 36 and 42.75 mm at 9 years for model C8 overlaid on the trajectories of observed values over time.

Figure 6 shows the relative error in accumulated cork thickness for complete year predictions by the number of complete years after cork extraction and length of the rotation period (from 9 to 12). When the debarking period is 9 years (432 observations) the relative error of the model prediction for any complete year after cork harvest is very low, less than 18% from the third year onwards. The reliability of the prediction decreases when the length of the rotation period increases, mainly due to the lower number of observations. In any case, from the fifth year onwards, the accumulated cork thickness in complete year predictions has a reliability of more than 80% for any debarking period.

Discussion

In this study, a base age invariant cork growth model was developed using the generalised algebraic difference approach (Cieszewski 1994; Cieszewski and Bailey 2000). Among the different base age invariant parameter estimation techniques available, we used the nested iterative procedure of the stochastic regression approach (Tait et al. 1988; Cieszewski et al. 2000). As shown in Tables 5, 6, the dynamic models derived by the generalised algebraic difference approach (GADA) described the data as well as the algebraic difference approach (ADA). A similar result was reported by Diéguez-Aranda et al. (2006) when developing a site index model for Loblolly pine plantations in USA. The GADA formulation derived by Krumland and Eng (2005) from the Richards model by considering a_1 and a_3 as related to site productivity (C8) performed slightly better and was therefore selected for describing cork growth in Spanish cork oak forests. This dynamic equation provides polymorphic curves with variable asymptotes. The predictive ability of this model was very high, as

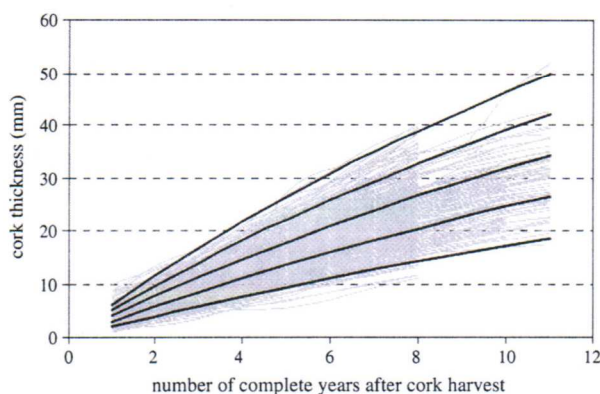


Fig. 5 Cork growth curves for accumulated cork thickness in complete years of 15.75, 22.5, 29.25, 36 and 42.75 mm at 9 years for model C8 overlaid on the trajectories of observed values over time

evidenced by the statistics obtained in the multiple cross-validation phase (Table 6).

In recent years, programs for sampling cork production have been put into action in Spain and Portugal. These programs are offered as a service to the land owners by the regional authorities in order to evaluate cork quality in the year prior to debarking by taking cork samples. It is very important to know the accumulated cork thickness in complete years to avoid a high percentage of waste and to decide on whether or not to delay the cork harvest. The model proposed in this paper may serve to improve these programs by providing information on cork thickness at any given year during the cork rotation. In the fourth year after cork extraction, the reliability of the predictions was nearly 80% (Fig. 6) for any debarking period, increasing to nearly 90% by the eighth year. The decrease in predictive ability when the debarking period is greater than 9 years is due to the fact that there are fewer observations from samples taken where the cork rotation was 11 or 12 years.

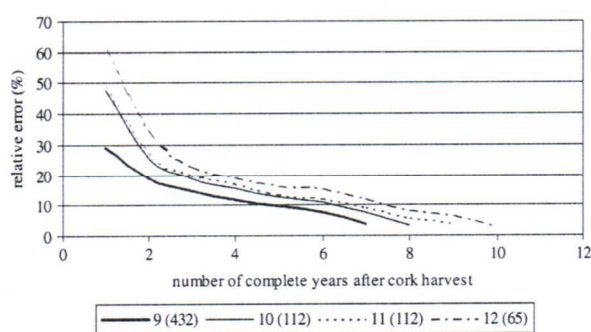


Fig. 6 Relative error in accumulated cork thickness for complete year predictions using the selected model C8 by number of complete years after cork harvest and by length of the debarking period. The number of observations available for each debarking period is given in brackets

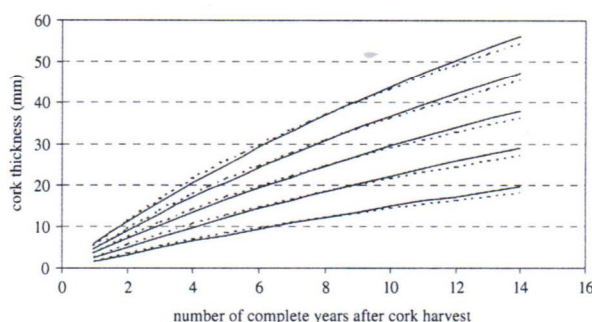


Fig. 7 Comparison between cork growth curves for a base age of nine years generated by the model developed by Tome et al. (2001)—dashed line—and the model proposed in this paper—solid line

In addition, this model could be used to define cork quality classes in order to classify cork stands in terms of their potential for producing cork suitable for the stopper industry.

In Fig. 7, the cork growth curves obtained are compared with those obtained by Tomé et al. (2001) for open cork oak woodlands in Portugal. Both models were plotted using a reference age of 9 years. Although the models were built using different approaches and each has different characteristics, the behaviour of the models tends towards greater similarity as the reference age increases, especially when the reference age is near to the end of the debarking period. The evaluation of accumulated cork thickness in complete years in order to decide on whether or not to delay the cork harvest is usually made the year before debarking, that is, from the eighth year onwards. Consequently, this is when the model becomes most useful. It might be interesting in future studies to evaluate the possibility of developing a single cork growth model which would be viable for cork oak woodlands as well as forests in both countries.

In conclusion, cork thickness is an important variable which defines the usability and value of the cork for industrial purposes. For this reason, a tool is required to help fix the most suitable rotation length in order to obtain cork planks suitable for stopper production. The cork growth model developed in this study allows us to estimate the final accumulated growth in complete years from the second year after cork extraction with more than 80% reliability, or nearly 90% if the prediction is made in the second half of the rotation.

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